Solution: Exam "Tax Policy" - January 2011

Marko Koethenbuerger

Exercise 1

Consider the tax is levied on the use of capital in sector 2. Two effects can be distinguished:

1. **Substitution effect**: It measures how a shift in the input mix (given output level) influences the interest rate.

- Tax on capital in sector 2, K_2, shifts production in Sector 2 away from capital K to labor L; so aggregate demand for K goes down.
- Because total amount of capital K is fixed, r falls $\rightarrow$ capital bears some of the burden.

2. **Output effect**: It measures the change in the interest rate following output changes in the two sectors (for a given input mix).

- Tax on capital K_2 implies that sector 2 output becomes more expensive relative to sector one
- Therefore output demand shifts toward sector 1 with corresponding adjustments in capital demand in both sectors.
- *Case 1:* The capital intensity in sector 1 is smaller than in sector 2, $K_1/L_1 < K_2/L_2$.
  - Sector 1 is less capital intensive so aggregate demand for K goes down.

- Output effect reinforces substitution effect: K bears the burden of the tax.
- Capital bears more than 100% of the burden if output effect is sufficiently strong.

- *Case 2:* $K_1/L_1 > K_2/L_2$
  - Sector 1 is more capital intensive, aggregate demand for K increases.
  - Substitution and output effects have opposite signs; labor may bear some or all the tax.
  - If output effect is sufficiently strong, the interest rate increases. Labor forced to bear more than 100% of incidence of capital tax in sector 2.

Exercise 2

2.1.

- Lagrangian for household's maximization problem:

$$\mathcal{L} = U(c_1, .., c_N, l) + \alpha(wl - (p_1 c_1 + .. + p_N c_N)),$$

  where α is the Lagrange multiplier associated with the household budget constraint. Note, the damage term $d(c_N)$ does not show up in the optimization problem because it is neglected by the household.

- First order condition:

$$U_{c_i} = \alpha p_i \text{ and } -U_l = \alpha w.$$

2.2.

The household's optimization problem yields demand functions $c_i^*(p)$, labor supply function $l^*(p)$ and indirect utility function $V(p)$ where $p = (w, p_1, .., p_N)$. Let's consider the effect of a higher tax (let's choose the tax on good i) on household utility $V(p)$. By the envelope theorem (Roy's identity) we have

$$\frac{\partial V}{\partial p_i} = -\alpha c_i^*.$$

Note, $\partial V/\partial p_i = \partial V/\partial t_i$ since $p_i = 1 + t_i$.

2.3.

Government solves the maximization problem

$$\max V(p) - d(c_N^*)$$

subject to the revenue requirement

$$\sum_{i=1}^{N} t_i c_i^*(p) = R$$

The Lagrangian for the government is:

$$\mathcal{L}_G = V(p) - d(c_N^*) + \lambda[\sum_i t_i c_i^*(p) - R],$$

where λ is the Lagrange multiplier associated with the government budget constraint.

Note, $\partial p_i/\partial t_i = 1$ since $p_i = 1 + t_i$. Thus, differentiating with respect to t_i is equivalent to differentiating with respect to p_i. Both approaches are fine. As a matter of choice, differentiating with respect to p_i yields

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = \frac{\partial V}{\partial p_i} - d' \frac{\partial c_N^*}{\partial p_i} + \lambda \left[c_i^* + \sum_{j=1}^{N} t_j \frac{\partial c_j^*}{\partial p_i} \right] = 0.$$

The first-order condition aligns the change in household utility due to a higher tax rate, $\partial V/\partial p_i - d' \partial c_N^*/\partial p_i$, to the rise in revenues expressed in utility changes, $-\lambda(c_i^* + \sum_{j=1}^{N} t_j \partial c_j^*/\partial p_i)$.

2.4.

The first order condition

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = \frac{\partial V}{\partial p_i} - d' \frac{\partial c_N^*}{\partial p_i} + \lambda \left[c_i^* + \sum_{j=1}^{N} t_j \frac{\partial c_j^*}{\partial p_i} \right] = 0.$$

can be rewritten as

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = \frac{\partial V}{\partial p_i} - d' \frac{\partial c_N^*}{\partial p_i} + \lambda \left[c_i^* + \sum_{j=1}^{N-1} t_j \frac{\partial c_j^*}{\partial p_i} + t_N \frac{\partial c_N^*}{\partial p_i} \right] = 0.$$

Combining the terms $\partial c_N^*/\partial q_i$, we get

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = \frac{\partial V}{\partial p_i} + \lambda \left[c_i^* + \sum_{j=1}^{N-1} t_j \frac{\partial c_j^*}{\partial p_i} + \left(t_N - \frac{d'}{\lambda} \right) \frac{\partial c_N^*}{\partial p_i} \right] = 0.$$

Now, define the adjusted tax rate on good N as $\tilde{t}_N = t_N - d'/\lambda$. Using the adjusted tax rate, the first-order condition reduces to the one in the absence of externalities:

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = \frac{\partial V}{\partial p_i} + \lambda \left[c_i^* + \sum_{j=1}^{N-1} t_j \frac{\partial c_j^*}{\partial p_i} + \tilde{t}_N \frac{\partial c_N^*}{\partial p_i} \right] = 0.$$

Without externalities the optimal tax rate for commodity N is $\tilde{t}_N$. With externalities it is $t_N = \tilde{t}_N + d'/\lambda$. This shows that the only adjustment in the optimal commodity tax system which the damage term requires is that the tax on the good which generates the negative externality is modified. That is, the tax rate on good N is increased to internalize the damage the consumption of good N causes. All other taxes are not affected.

Exercise 3

3.1.

Inserting $\epsilon(w) = \bar{\epsilon}$ and the functions $f(w)$ and $F(w)$ into the right-hand side of the formula given in the exercise gives:

$$\frac{T'(w)}{1 - T'(w)} = \frac{1 + \bar{\epsilon}}{\bar{\epsilon}} \frac{1}{a}$$

Solving for $T'(w)$, we get

$$T'(w) = \frac{1 + \bar{\epsilon}}{1 + (1 + a)\bar{\epsilon}}$$

3.2.

Income and productivity follows the same distribution, the value of a must be the same for the income and productivity distribution. Note, $[1 - H(z)]/zh(z) = 1/a = 0.5$ on average for high incomes. That is, the average value of a which follows from the graph for high incomes is 2.

3.3.

Plugging in $\bar{\epsilon} = 0.2$ and $a = 2$ in the formula for the marginal tax rate

$$T'(w) = \frac{1 + \bar{\epsilon}}{1 + (1 + a)\bar{\epsilon}}$$

yields an optimal marginal tax rate for high incomes of $3/4$.

Hint: If $a = 2$ is not taken from 3.2. and $T'(w)$ is computed for a general a, then this is also correct.

Exercise 4

4.1.

Government tax revenues in period t are

$$R_t = \tau^k r_t K_t + \tau^l w_t L_t.$$

4.2.

Expanding the revenue term

$$R_t = \tau^k r_t K_t + \tau^l w_t L_t.$$

yields

$$R_t = (1 - 1 + \tau^k) r_t K_t + (1 - 1 + \tau^l) w_t L_t.$$

Using the definition of $\bar{r}_t$ and $\bar{w}_t$ (as given in the exercise), we have

$$R_t = (r_t - \bar{r}_t) K_t + (w_t - \bar{w}_t) L_t.$$

4.3.

The Lagrangian of the government decision problem is

$$\mathcal{L} = V_1 + \sum_{t=1}^{\infty} \lambda_t (F_t(K_t, L_t) + K_t - C_t - G_t - K_{t+1}) + \sum_{t=1}^{\infty} \mu_t (b_{t+1} - (1+\bar{r}_t) b_t - \bar{r}_t K_t - \bar{w}_t L_t + F_t(K_t, L_t) - G_t).$$

λ_t and μ_t are the Lagrange multipliers associated with the resource constraint and public budget constraint in period t.

4.4.

The first-order condition for K_{t+1} is

$$-\lambda_t + \lambda_{t+1}\left(\frac{\partial F_{t+1}(K_{t+1}, L_{t+1})}{\partial K_{t+1}} + 1\right) + \mu_{t+1}\left(-\bar{r}_t + \frac{\partial F_{t+1}(K_{t+1}, L_{t+1})}{\partial K_{t+1}}\right) = 0.$$

Note, with competitive markets firms use capital until

$$\frac{\partial F_{t+1}(K_{t+1}, L_{t+1})}{\partial K_{t+1}} = r_{t+1}.$$

Thus, the first-order condition is

$$-\lambda_t + \lambda_{t+1}(r_{t+1} + 1) + \mu_{t+1}(-\bar{r}_t + r_{t+1}) = 0.$$

4.5.

Along a stationary trajectory, the interest rates (before and after tax) stay constant. Along a stationary trajectory, the discounted Lagrange multipliers λ_{t+1} and μ_{t+1} are related to their values in each period t by

$$\tilde{\lambda} = (1+\bar{r})^{t+1}\lambda_{t+1} \text{ and } \tilde{\mu} = (1+\bar{r})^{t+1}\mu_{t+1}.$$

Thus, the first-order condition reduces to

$$-\tilde{\lambda}(1+\bar{r})^{-t} + \tilde{\lambda}(1+\bar{r})^{-(t+1)}(r+1) + \tilde{\mu}(1+\bar{r})^{-(t+1)}(-\bar{r}+r) = 0.$$

Rearranging, we get

$$(\tilde{\lambda} + \tilde{\mu})(r - \bar{r}) = 0$$

Since the Lagrange multipliers $\tilde{\lambda}$ and $\tilde{\mu}$ are positive, we have $r = \bar{r}$. This implies a zero tax on capital, $\tau^k = 0$.

Intuition:

- Deadweight loss of taxation rises with square of tax rate.

- With non-zero capital tax, we have an infinite price distortion between C_0 and C_t as $t \to \infty$

- Undesirable to have such large distortions on some decision margin.